Int. J. Heat Mass Transfer. Vol. 33, No. 2, pp. 393–400, 1990
Printed in Great Britain

0017-9310/90 \$3.00+0.00

Dissipation of turbulent kinetic energy in buoyant free shear flows

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(*Received* 30 *June* 1988 *and in final form* 10 *March* 1989)

Abstract—A new scaling of the mean rate of turbulent kinetic energy dissipation is proposed for application to free shear flows. The advantage of the proposed scaling is that it permits a direct comparison of the dissipation rate in flows driven by completely different mechanisms (e.g. inertia or buoyancy forces) and of different flow configurations. The same scaling, when applied to pipe flow suggests that the non-dimensional parameter so formed is analogous to a 'friction factor' for this motion. Application of this development to the motion of buoyant jets and plumes suggests that the mean rate of dissipation in buoyancy driven plume flows may be twice that for jets and is associated with the larger rate of entrainment in such flows.

1. INTRODUCTION

THE RATE at which kinetic energy is dissipated is an important parameter in the description of any fully developed turbulent flow where, according to the model proposed by Kolmogorov, energy is transferred in a cascade process from larger to smaller scales. This process appears essentially inviscid down to the equilibrium range of scales where the viscosity ν becomes important and, under its action, kinetic energy is dissipated into heat. At equilibrium the rate of energy supply from the mean flow to the largest scales is therefore equal to the rate of kinetic energy dissipation. It should be noted that while the dynamics of the large structures of the turbulent flow are governed by the overall geometry and driving forces, the dynamics of the smallest structures in this model are governed only by the viscosity and the dissipation rate. The value of the strain rate $(\varepsilon/\nu)^{1/2}$ defining the Kolmogorov range of scales of the flow, becomes a critical parameter in the study of processes that occur at these small scales. These, for example, would include the coagulation of suspended particles in a moving fluid, the formation of droplets in a cloud, the mechanism of mixing and product formation in chemically reacting shear flows, etc. The rate at which naturally occurring flows are dissipating energy is important in engineering problems concerned with the disposition of particles in an atmospheric or water environment. Flows of this nature are often characterized by the presence of buoyancy as a driving force of motion. As was pointed out by List [1], the research literature concerning buoyancy driven laboratory flows is deficient compared to inertia driven flows in general. While, for example, measurements of the kinetic energy dissipation rate have been made in the plane and round jets, shear layers, wakes and boundary layers, similar measurements in buoyant plumes have not been reported. This may be partly attributed to the increased experimental difficulty associated with measurements in such buoyancy driven flows. On the other hand Turner [2] pointed out the large energy deficiency in self-preserving convective plumes or thermals.

The rate of dissipation of total kinetic energy in a turbulent flow (see for example Townsend [3]) can be expressed as

$$\varepsilon = \nu\left[U_i\frac{\partial^2 U_i}{\partial x_j^2} + \overline{u_i\frac{\partial^2 u_i}{\partial x_j^2}}\right].$$

In free turbulent flows the mean velocities U_i vary smoothly across the flow, therefore spatial derivatives are negligible compared to the spatial derivatives of the instantaneous velocity fluctuations u_i and can be neglected. Then the kinetic energy dissipation rate is approximated by

$$\varepsilon = \frac{\nu}{2}\overline{\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2}. \tag{1}$$

Thus, a direct measurement of the dissipation rate at a point in the flow would involve measurement of all terms in equation (1). This has not been achieved because of the obvious experimental difficulties associated with such a measurement. The usual approach taken involves (i) making an assumption of isotropy relevant to the particular flow in order to reduce the number of terms to be measured and (ii) invoking Taylor's frozen turbulence hypothesis to replace the spatial derivatives with temporal derivatives. Sometimes other terms in the equation for the turbulent kinetic energy are also measured and a turbulence kinetic energy budget is obtained. However, the pressure velocity correlation term has not been measured yet, hence this equation cannot be used to check the accuracy of these two assumptions. Pointwise estimates of ε in plane and round turbulent jets have been

NOMENCLATURE

$b(x)$	half width	U_0	jet velocity at the exit.
d	jet diameter at the exit		
$D(x)$	total dissipation of energy per unit time (equation (12))		
		Greek symbols	
E	flux of mean flow kinetic energy across a horizontal plane (equation (11)), $E(x)$	α	parameter equal to 2 for a flow with plane symmetry and equal to 3 for a flow with axial symmetry
$E_e(x)$	flux of kinetic energy of the entrained fluid (equation (14))	β_0	buoyancy flux
$E_t(x)$	flux of turbulent kinetic energy across a horizontal plane at x (equation (14))	ε	rate of dissipation of total kinetic energy
		$\hat{\varepsilon}$	averaged kinetic energy dissipation rate over a flow volume V
f	friction coefficient in the Darcy–Weisbach equation	E	non-dimensional number, $\hat{\varepsilon}\Lambda^{\alpha}/E$
		Λ	width of turbulent flow—ambient fluid boundaries, $\Lambda(x)$
u_i	turbulent fluctuation along the x_i-axis		
$U_e(x)$	entrainment velocity	ν	kinematic viscosity
U_i	mean velocity along the x_i-axis	τ	mean interfacial stress
$U_m(x)$	mean axial velocity	ϕ	experimental coefficient.

performed by Heskestad [4], Bradbury [5], Wygnanski and Fiedler [6], Gutmark and Wygnanski [7] and Antonia *et al.* [8]. There appear to be no such measurements in buoyant jets and plumes. However, reasonable estimates of the kinetic energy dissipation rate may be obtained from the available experimental data on such flows. To this end, we introduce $\hat{\varepsilon}$, the averaged kinetic energy dissipation rate over a flow volume V defined as

$$\hat{\varepsilon} = \frac{\int_V \varepsilon(x,y)\,\mathrm{d}V}{V}. \tag{2}$$

Time exposure photographs and measurements in both jets and plumes indicate that the region where jet fluid may be found is separated from the ambient fluid region by approximately linear boundaries (see Fig. 1) $\Lambda(x)$ which may be well approximated by

$$\Lambda(x) = \phi x \tag{3}$$

where $\phi \simeq 0.5$ for round or plane jets and plumes.

Specifying the integrating volume V to be the infinitesimal volume dV bounded by the planes x and x dx (see Fig. 1) we define the averaged kinetic energy dissipation $\hat{\varepsilon}$ (per unit depth) for plane jets and plumes by

$$\hat{\varepsilon}(x)\Lambda(x)\,\mathrm{d}x = \int_x^{x+\mathrm{d}x}\int_{-\Lambda(x)/2}^{\Lambda(x)/2} \varepsilon(x,y)\,\mathrm{d}x\,\mathrm{d}y \tag{4}$$

and for round jets and plumes by

$$(\hat{\varepsilon})(\pi\Lambda^2/4)\,\mathrm{d}x = \int_x^{x+\mathrm{d}x}\int_{-\Lambda(x)/2}^{\Lambda(x)/2} 2\pi r\varepsilon(x,r)\,\mathrm{d}x\,\mathrm{d}r.$$

Estimates of $\hat{\varepsilon}$ will be obtained for the turbulent jet and plume using experimental measurements of the mean and turbulent flow quantities.

2. SCALING LAWS

Friehe *et al.* [9] derived the following relation between the kinetic energy dissipation rate ε along the axis of a round jet, the exit diameter d, the velocity at the jet exit U_0 and the axial distance x from the flow origin:

$$\varepsilon(x,0)\,d/U_0^3 = C_1(x/d)^{-4}. \tag{5}$$

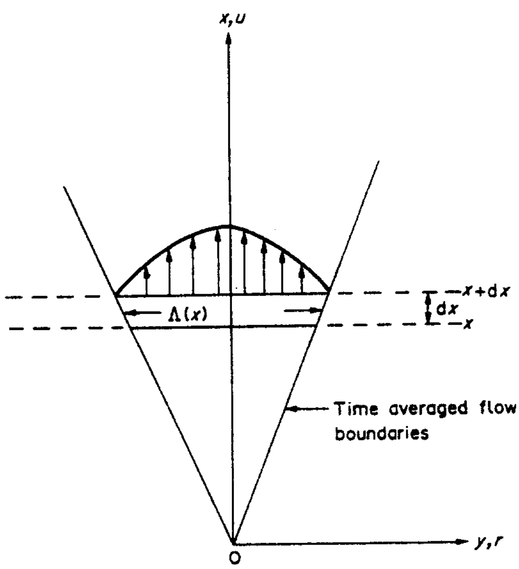

FIG. 1. Definition of the flow width $\Lambda(x)$. The origin of the jet (or plume) is at $x = y = 0$.

Similarly, Antonia *et al.* [8] proposed for a plane jet with slot width d, the relation

$$\varepsilon(x,0)\, d/U_0^3 = C_2(x/d)^{-5/2}. \qquad (6)$$

Although the above scaling for the dissipation rate is successful in reducing experimental results in similar flow geometries, the parameter $\varepsilon d/U_0^3$ seems to have limited physical significance. In particular, this scaling is not directly applicable to a buoyant jet or a plume of similar geometry. On physical grounds, one would expect that the dissipation rate at a certain axial location for both jets and plumes to depend on local flow scales and not on initial parameters, such as the orifice diameter. In view of recent developments in the understanding of the importance of local integral quantities in jet dynamics (see for example List [1], it seems reasonable to scale the averaged dissipation rate $\hat{\varepsilon}$, with the local kinetic energy flux E and the local flow width Λ. Then, on dimensional grounds, we obtain the non-dimensional number

$$\mathrm{E} = \frac{\hat{\varepsilon}\Lambda^{\alpha}}{E} \qquad (7)$$

where $\alpha = 2$ for a flow with plane symmetry and $\alpha = 3$ for a flow with axial symmetry. Note that for plumes and other shear flows where the flow development depends only on the distance x from the flow origin, we have

$$\mathrm{E} = \frac{\hat{\varepsilon}(x)\Lambda^{\alpha}(x)}{E(x)}. \qquad (8)$$

The advantage of the proposed scaling is that it permits a direct comparison of the dissipation rate in flows driven by completely different mechanisms (e.g. inertia or buoyancy forces) and of different flow configurations (jets, shear layers, wakes, wall jets, boundary layers, etc.).

An interesting result regarding the physical interpretation of the non-dimensional averaged dissipation rate E can be gained by calculating the dissipated energy per unit time in a circular pipe of diameter Λ. If f is the friction coefficient (in the Darcy–Weisbach equation) one may easily find that

$$\mathrm{E} = \frac{\hat{\varepsilon}(x)\Lambda^3}{E(x)} = \frac{4f}{\pi}. \qquad (9)$$

It can be seen from this example that in bounded turbulent flows (when buoyancy is not present), the non-dimensional averaged energy dissipation rate defined by equation (7) is equal to the friction coefficient multiplied by a constant depending only on the geometry of the flow. Turning this around, one may be tempted to interpret the parameter E for free turbulent flows, as a gross interfacial 'friction coefficient'.

3. ESTIMATES FOR THE KINETIC ENERGY DISSIPATION IN JETS AND PLUMES

To obtain estimates for the averaged dissipation rate $\hat{\varepsilon}$ in buoyant jets from available experimental data, we will perform an overall energy balance in a suitably chosen flow volume (see Fig. 1) using the available experimental data for the mean and turbulent quantities. The ambient fluid is assumed of uniform density ρ_a and motionless, the initial velocity is U_0; d the width of the flow at the origin. It is easy to find that the requirement for overall conservation of kinetic energy flux in the flow region between two horizontal cross sections at $x = 0$ and x (see Turner [2]) gives

$$E_0 + \beta_0 x = D(x) + E(x) + E_t(x) + E_e(x) \qquad (10)$$

where E_0 is the flux of input kinetic energy, β_0 the buoyancy flux, which is constant for an unstratified environment, $\beta_0 x$ the work done by buoyancy forces per unit time between two horizontal cross sections at $x = 0$ and x, $E(x)$ the flux of mean flow kinetic energy across a horizontal plane

$$E(x) = \frac{1}{2}\int_{-\Lambda(x)/2}^{\Lambda(x)/2} (2\pi y)^i U^3(x,y)\,\mathrm{d}y$$

$$\begin{cases} i = 0 & \text{for plane flow geometry} \\ i = 1 & \text{for round jet or plume} \end{cases} \qquad (11)$$

$D(x)$ the total dissipation of energy per unit time in the buoyant jet between the two horizontal cross sections at $x = 0$ and x

$$D(x) = \int_0^x \int_{-\Lambda(x)/2}^{\Lambda(x)/2} (2\pi y)^i \varepsilon(x,y)\,\mathrm{d}x\,\mathrm{d}y \qquad (12)$$

$E_t(x)$ the flux of turbulent kinetic energy across a horizontal plane at x

$$E_t(x) = \frac{1}{2}\int_{-\Lambda(x)/2}^{\Lambda(x)/2} (2\pi y)^i U(x,y) q^2\,\mathrm{d}y \qquad (13)$$

$E_e(x)$ the flux of kinetic energy of the entrained fluid between the two horizontal planes at $x = 0$ and x

$$E_e(x) = \int_0^x \int_{-\Lambda(x)/2}^{\Lambda(x)/2} (2\pi y)^i U_e^3(x)\,\mathrm{d}x\,\mathrm{d}y \qquad (14)$$

$$\bar{q}^2 = \overline{u'^2} + \overline{v'^2} + \overline{w'^2} = \text{turbulence intensity} \qquad (15)$$

$$U_e(x) = \text{entrainment velocity}$$

$$U(x,y) = \text{axial velocity} = U_M(x)\exp[-\ln 2(y/b(x))^2] \qquad (16)$$

$U_M(x)$ the velocity along the axis of the flow and $b(x)$ the half width.

In both round or plane buoyant jets the induced flow kinetic energy $E_e(x)$ is a negligible fraction ($<3\%$) of the mean kinetic energy flux $E(x)$ and of the total dissipation $D(x)$. Combining equations (2) and (10) we find that the average dissipation rate $\hat{\varepsilon}(x)$ in the infinitesimal plume volume bounded by two horizontal planes at x and x dx is given by

$$\hat{\varepsilon}\,\mathrm{d}V = \beta_0 - \frac{\mathrm{d}}{\mathrm{d}x}[E(x) + E_t(x)]. \qquad (17)$$

Table 1.

Type of flow (1)	Centreline axial velocity $U_M(x)$ (2)	Half width, $b(x)$ (3)	Jet boundaries, $\Lambda(x)$ (4)	Flux of mean kinetic energy, $E(x)$ (5)	Flux of turbulent kinetic energy, E_t (6)	Experimental constants (7)	Non-dimensional dissipation, E (8)	The experimental constants are found in the corresponding references (9)
Plane jet	$U_0^2/U_M^2 = C_{2j}(x/d)$	$b(x) = K_{2j}x$	$\Lambda(x) = \phi_{2j}x$	$E(x) = \sqrt{\left(\frac{\pi}{12 \ln 2}\right)} K_{2j}C_{2j}^{3/2} \times (x/d)^{-1/2}\, dU_0^3$	$E_t \simeq a_{2j}E(x)$	$a_{2j} = 0.14$ $a_{2j} = 0.5$	$E_{2j} = \frac{\hat{\varepsilon}\Lambda^2}{E} = \frac{(1+a_{2j})}{2}\phi_{2j} = 0.29$	Heskestad [4] Kotsovinos and List [10]
Plane plume	$U_M(x) = C_{2p}\beta^{1/3}$	$b = K_{2p}x$	$\Lambda = \phi_{2p}x$	$\sqrt{\left(\frac{\pi}{12 \ln 2}\right)} K_{2p}C_{2p}^3\beta x$	$E_t = a_{2p}E(x)$	$C_{2p} = 1.8$ $K_{2p} = 0.097$ $a_{2p} = 0.3$ $\phi_{2p} = 0.5$	$E_{2p} = \frac{\hat{\varepsilon}\Lambda^2}{E} = \left[\sqrt{\left(\frac{12 \ln 2}{\pi}\right)} \frac{1}{K_{2p}C_{2p}^3} - (1+a_{2p})\right]\phi_{2p} = 0.79$	Kotsovinos [11] Chen and Rodi [12]
Plane bubble plume	$U_M = C_{2b}\beta^{1/3}$	$b = K_{2b}x$	$\Lambda = \phi_{2\beta}x$	$E(x) = \sqrt{\left(\frac{\pi}{12 \ln 2}\right)} K_{2b}C_{2b}^3\beta x$	$E_t = a_{2b}E(x)$	$K_{2b} = 0.09$ $C_{2b} = 1.4$ $\phi_{2b} = 0.45$	$E_{2b} = \frac{\varepsilon\Lambda^2}{E} = \left[\sqrt{\left(\frac{12 \ln 2}{\pi}\right)} \frac{1}{K_{2b}C_{2b}^3} - (1+a_{2b})\right]\phi_{2b} = 2.29$	Wilkinson [13]
Mixing layer—one stream is not moving			$\Lambda = \phi_{2m}x$				$E_m = \frac{\hat{\varepsilon}\Lambda^2}{E(x)} = \phi_{2m} = 0.16$	Rodi [14]

Round jet	$U_0/U_M(x) = C_{3j}(x/d)$	$b = K_{3j}x$	$\Lambda = \phi_{3j}x$	$E(x) = \frac{\pi}{6 \ln 2} K_{3j}^2 C_{3j}^{-3} U_0^3 x^{-1} d^3$	$E_t(x) = a_{3j}E(x)$	$a_{3j} = 0.16$ $\phi_{3j} = 0.5$	$\mathrm{E}_{3j} = \frac{\hat{\varepsilon}\Lambda^3}{E} = \frac{4(1+a_{3j})\phi_{3j}}{\pi} = 0.70$	Fischer *et al.* [15]
Round plume	$U_M = C_{3p}\beta^{1/3}x^{-1/3}$	$b = K_{3p}x$	$\Lambda = \phi_{3p}x$	$E(x) = \frac{\pi C_{3p}^3 K_{3p}^2 \beta x}{6 \ln 2}$	$E_t = a_{3p}E$	$C_{3p} = 3.85$ $K_{3p} = 0.087$ $a_{3p} = 0.16$ $\phi_{3p} = 0.5$	$\mathrm{E}_{3p} = \frac{\hat{\varepsilon}\Lambda^3}{E} = \frac{4}{\pi}\left[\frac{6 \ln 2}{\pi C_{3p}^3 K_{3p}^2} - (1+a_{3p})\right] = 1.25$	Papanikolaou [16]
Couette flow	U = constant		Λ = constant	$E = \frac{1}{2}\Lambda U^3$	$E_t = a_{2c}E$		$\mathrm{E}_{2c} = \frac{\hat{\varepsilon}\Lambda^2}{E} = f$ f = friction coefficient	
Flow in a circular pipe	U = constant		Λ = constant	$E = \frac{1}{2}(\pi\Lambda^2/4)U^3$	$E_t = a_{3c}E$		$\mathrm{E}_{3c} = \frac{\hat{\varepsilon}\Lambda^3}{E} = \frac{4f}{\pi}$ f = friction coefficient (Darcy)	
Lock exchange flow (stably stratified flow)	U = velocity of salt water front		Λ = total water depth	$E = \Lambda U^3$			$\mathrm{E} = \frac{\hat{\varepsilon}\Lambda^2}{E} = 8\lambda$ λ = interfacial friction coefficient	Abraham and Eysink [17]

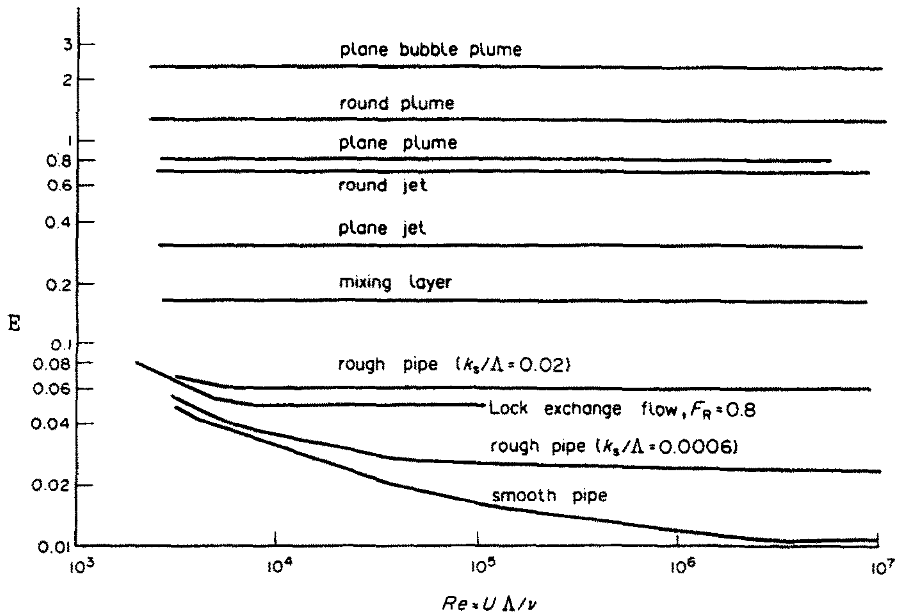

FIG. 2. Generalized friction coefficient E (= non-dimensional dissipation rate of kinetic energy) as a function of the Reynolds number Re. For pipe flow, Λ is equal to the pipe diameter D; k_s is the pipe's roughness. This figure can be considered as an extension of the well-known Moody diagram to free turbulent flows. Also plotted for comparison are the results of Abraham and Eysink [17] for the energy dissipated due to interfacial shear in lock exchange stratified flow for an internal Froude number of 0.8.

It should be noted that equation (17) is valid for a turbulent plane or round buoyant jet of any Richardson number, and for free turbulent shear flows.

Existing experimental results will be used to obtain numerical estimates for $E(x)$ and $E_t(x)$ for jets and plumes.

The calculations are presented in Table 1, and the results are plotted in Fig. 2.

The results presented in Table 1 indicate that the non-dimensional averaged dissipation rate $\mathrm{E} = \bar{\varepsilon}\Lambda^x/E(x)$ is much larger for plumes than for jets, mainly for the plane geometry, i.e. given two flows, one driven by inertia forces and one by buoyancy forces, with equal width Λ and equal mean kinetic energy flux $E(x)$, the kinetic energy dissipation is larger for the flow driven by the buoyancy forces.

This implies that the fictitious, gross interfacial 'friction coefficient' E is larger for plumes than for jets. Note that the mean interfacial stress between jet (or plume) fluid (moving with a local mean velocity U_m) and ambient (motionless) fluid can be defined as

$$\tau = \mathrm{E}\rho U_m^2$$

and it can be seen to be much larger for the plume than for the jet. This increase is consistent with the doubling of the entrainment coefficient in the transition from pure plane jet to pure plane plume, and with the 5/3 increase of the entrainment coefficient in the transition from pure round jet to pure round plume [15]. The increase E from jets to plumes is larger for the plane geometry than for the axisymmetric and consistent with the similar increase of corresponding entrainment coefficient.

It is worthwhile to point out that the arithmetic value of the non-dimensional average kinetic energy dissipation E depends strongly on the mean flow parameters (which can be determined with reasonable accuracy) and is only moderately sensitive to the turbulence structure (parameter a_{2j} or a_{2p}, etc., see Table 1).

For example in Table 1 we estimated that $a_{2p} \simeq 0.3$ and we found $\mathrm{E}_{2p} \simeq 1.05$. Assuming 100% error in estimating a_{2p}, i.e. assuming $a_{2p} \simeq 0.15$, we find $\mathrm{E}_{2p} \simeq 1.13$, i.e. a 100% decrease of the flux of turbulent kinetic energy gives only a 7% increase of the dimensionless dissipation E_{2p}.

4. CONCLUSIONS

The finding of this paper that the non-dimensional kinetic energy dissipation rate, defined as $\mathrm{E} = \bar{\varepsilon}\Lambda^x/E$, depends on the buoyancy structure of the flow is interesting.

Higher values of the kinetic energy dissipation rate under buoyant instability, which is by its definition related to larger instantaneous velocity gradients, are probably due to the increased spatial density inhomogeneities that exist in buoyancy driven flows. In the Eulerian description, this corresponds to an increase in the intensity of density (or temperature) and velocity fluctuations.

An analysis similar to the one that led to Table 1 is suitable for various other shear flow configurations with or without the presence of buoyancy. We believe that it offers an alternative approach to the (otherwise complex) task of the evaluation of the kinetic energy dissipation rate—once the mean velocity profile, and the turbulence intensity profile have been determined with sufficient accuracy. In a flow with plane or axial symmetry, the use of the averaged dissipation rate $\hat{\varepsilon}$ is equivalent to an assumption of a 'top-hat' profile for the distribution of ε, with a base equal to the width of the flow and height $\hat{\varepsilon}$. Our belief in the usefulness of this assumption is strengthened by the findings of Gutmark and Wygnanski [7] regarding the shape of the kinetic energy dissipation profile in a plane jet when the time average was taken in the turbulent zone. They found that this conditionally averaged profile was flat, indicating that the familiar bell-shaped lateral distribution of ε (reported for example in Bradbury [5]) is probably due to the intermittent presence of ambient entrained fluid in the jet flow field. It is of interest to note that the averaged dissipation rate $\hat{\varepsilon}$, can be computed directly from the pointwise measurements of the energy dissipation $\varepsilon(x, y)$ of Gutmark and Wygnanski [7] in the plane jet, yielding

$$\mathrm{E} = \frac{\hat{\varepsilon}\Lambda^2}{E(x)} = 0.26$$

which compares favorably with our calculations based on the velocity profiles ($=0.29$). The advocated non-dimensional parameter E enables a good estimate of the mean dissipation rate $\hat{\varepsilon}$ to be made in flows where no such measurements exist. Moreover, it is a convenient scaling, in that it permits a direct comparison of ε in different flows (as shear layers, wakes, boundary layers, etc.). For example, we estimated, using the method outlined that $\hat{\varepsilon}\Lambda^2/E = 0.16$ in a shear layer where one stream is not moving and $\hat{\varepsilon}\Lambda^2/E = 2.29$ for a plane bubble plume.

It is interesting to notice that the value of E for non-buoyant jets and mixing layers is strongly dependent of the spreading angle, and is independent of the rate of velocity decay.

The shift from $\varepsilon d/U_0^3$ to E suggests a dependence of ε on the local mean flow parameters and contradicts, to some extent, Kolmogorov's first hypothesis, which states that the eddies in the dissipation range are in a statistical sense universal and are independent of the mean parameters of the flow (this remark was pointed out by a referee of this paper). Monin and Yalgom [18] recognize that ε may depend on the properties of large scale motion. As they state, this is in fact the main reason why they have used the term 'quasi-equilibrium range' in the discussion of Kolmogorov theory and avoided speaking of 'universal equilibrium'.

While more pointwise conditionally averaged measurements of ε are needed to understand the mechanism of energy dissipation, the usefulness of an integral analysis that relates E to local integral quantities governing the evolution of the flow field should not be underestimated.

Acknowledgements—This paper was written while the author was a visiting research associate at Caltech (summer 1983) holding a NATO Research Grant. The author is grateful to Professor E. J. List and to Dr D. A. Papantoniou for many constructive and enlightening discussions and to the reviewers of this paper for their valuable comments. The financial support of NATO (research grant 117.80/81) is gratefully acknowledged.

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DISSIPATION DE L'ENERGIE CINETIQUE TURBULENTE DANS DES ECOULEMENTS CISAILLANTS DE FLOTTEMENT

Résumé—On propose une étude de la dissipation de l'énergie cinétique turbulente, pour application aux écoulements libres de cisaillement. L'avantage de la méthode proposée est qu'elle permet une comparaison directe du taux de dissipation dans des écoulements mus par des mécanismes complètement différents (comme les forces d'inertie et celles de flottement) et de configurations d'écoulement différentes. La même mise à l'échelle, lorsqu'elle est appliquée à l'écoulement en conduite, suggère que le paramètre adimensionnel ainsi formé est analogue à un "coefficient de frottement" pour ce mouvement. L'application de ce développement au mouvement des jets flottants et des panaches suggère que le taux moyen de dissipation dans l'écoulement de panache conduit par flottement peut être deux fois celui des jets et qu'il est associé à la plus grande vitesse d'entrainement dans de tels écoulements.

DISSIPATION TURBULENTER KINETISCHER ENERGIE IN DURCH AUFTRIEBSKRÄFTE ERZEUGTEN FREIEN SCHERSTRÖMUNGEN

Zusammenfassung—Es wird eine neue Möglichkeit zur Bestimmung der mittleren Dissipation der turbulenten kinetischen Energie für die Anwendung bei freien Scherströmungen vorgeschlagen. Der Vorteil dieser Art der Bestimmung ist, daß sie einen direkten Vergleich der Dissipation in Strömungen mit völlig unterschiedlichen Antriebsmechanismen (z. B. Trägheits- oder Auftriebskräfte) und unterschiedlicher Strömungsform zuläßt. Wird diese Bestimmungsmöglichkeit auf eine Rohrströmung angewendet, so ist der vorgeschlagene dimensionslose Parameter analog dem "Reibungsbeiwert" dieser Strömung. Die Anwendung auf einen Auftriebsstrahl und eine Auftriebsfahne zeigt, daß die mittlere Dissipation in einer Auftriebsfahne bis zu zweimal so hoch sein kann wie in einem Auftriebsstrahl. Dies wird durch den stärkern Mitreißeffekt hervorgerufen.

ДИССИПАЦИЯ ТУРБУЛЕНТНОЙ КИНЕТИЧЕСКОЙ ЭНЕРГИИ В СВОБОДНОКОНВЕКТИВНЫХ СДВИГОВЫХ ПОТОКАХ

Аннотация—Предложено новое определение масштаба средней скорости диссипации турбулентной кинетической энергии для случаев свободных сдвиговых течений. Преимущество этого определения заключается в том, что оно позволяет сравнивать скорость диссипации в течениях, вызванных совершенно различными механизмами (т.е. инерцией или подъемными силами) и имеющими различную конфигурацию. Применительно к течению в трубах, данное определение масштаба предполагает, что указанный безразмерный параметр аналогичен "коэффициенту трения" для рассматриваемого течения. В случае же его применения к движению свободноконвективных струй и потоков предполагается, что средняя скорость диссипации в свободноконвективных восходящих потоках может вдвое превосходить эту же величину для струй и может быть связана с большей скоростью уноса в данных потоках.